1. The curve C with equation $y = k + \ln 2x$, where k is a constant, crosses the x-axis at the point $A\left(\frac{1}{2\mathrm{e}}, 0\right)$.

(a) Show that $k = 1$.

(2)

(b) Show that an equation of the tangent to C at A is $y = 2\mathrm{e}x - 1$.

(4)

(c) Complete the table below, giving your answers to 3 significant figures.

x	1	1.5	2	2.5	3
1 + ln 2x		2.10		2.61	2.79

(2)

(d) Use the trapezium rule, with four equal intervals, to estimate the value of

$$\int_1^3 (1 + \ln 2x) \quad \mathrm{d}x$$.

(4)
(Total 12 marks)

2. $$\mathrm{f}(x) = x + \frac{\mathrm{e}^x}{5}, \quad x \in \mathbb{R}.$$

(a) Find $\mathrm{f}'(x)$.

(2)

The curve C, with equation $y = \mathrm{f}(x)$, crosses the y-axis at the point A.

(b) Find an equation for the tangent to C at A.

(3)

(c) Complete the table, giving the values of $\sqrt{\left(x+\frac{e^x}{5}\right)}$ to 2 decimal places.

x	0	0.5	1	1.5	2
$\sqrt{\left(x+\frac{e^x}{5}\right)}$	0.45	0.91			

(2)

(d) Use the trapezium rule, with all the values from your table, to find an approximation for the value of

$$\int_0^2 \sqrt{\left(x+\frac{e^x}{5}\right)}\,dx .$$

(4)
(Total 11 marks)

3. A student tests the accuracy of the trapezium rule by evaluating I, where

$$I = \int_{0.5}^{1.5}\left(\frac{3}{x}+x^4\right)dx .$$

(a) Complete the student's table, giving values to 2 decimal places where appropriate.

x	0.5	0.75	1	1.25	1.5
$\frac{3}{x}+x^4$	6.06	4.32			

(2)

(b) Use the trapezium rule, with all the values from your table, to calculate an estimate for the value of I.

(4)

(c) Use integration to calculate the exact value of I.

(4)

(d) Verify that the answer obtained by the trapezium rule is within 3% of the exact value.

(2)

(Total 12 marks)

1.	(a)	$0 = k + \ln 2\left(\frac{1}{2\text{e}}\right) \Rightarrow 0 = k - 1 \Rightarrow k = 1$ (*)	M1 A1	2
		(Allow also substituting k = 1and x = $\frac{1}{2e}$ into equation and showing y = 0 and substituting k = 1 and y = 0 and showing x = $\frac{1}{2e}$.)		
	(b)	$\frac{\text{d}y}{\text{d}x} = \frac{1}{x}$	B1	
		At A gradient of tangent is $\frac{1}{1/2\text{e}} = 2\text{e}$	M1	
		Equations of tangent: $y = 2\text{e}\left(x - \frac{1}{2\text{e}}\right)$	M1	
		Simplifying to $y = 2\text{e}x - 1$ (*) cso	A1	4
	(c)	$y_1 = 1.69$, $y_2 = 2.39$	B1, B1	2
	(d)	$\int_1^3 (1 + \ln 2x)\text{d}x \approx \frac{1}{2} \times \frac{1}{2} \times (...)$	B1	
		$\approx \ldots \times (1.69 + 2.79 + 2(2.10 + 2.39 + 2.61))$ ft their (c)	M1 A1ft	
		≈ 4.7	A1	4
		accept 4.67		
				[12]
2.	(a)	Differentiating; $\mathbf{f'(x) = 1 + \frac{e^x}{5}}$	M1;A1	2
	(b)	A: $\left(0, \frac{1}{5}\right)$	B1	
		Attempt at $y - \text{f}(0) = \text{f}'(0)x$;	M1	
		$\boldsymbol{y - \frac{1}{5} = \frac{6}{5}x}$ or equivalent "one line" 3 termed equation	A1 ft	3
	(c)	**1.24, 1.55, 1.86**	B2(1,0)	2
	(d)	Estimate = $\frac{0.5}{2}$; (×) [(0.45 + 1.86) + 2(0.91 + 1.24 + 1.55)]	B1 M1 A1 ft	
		= **2.4275** $\left(\begin{matrix}\mathbf{2.428} \\ \mathbf{2.429}\end{matrix}, \ \mathbf{2.43}\right)$	A1	4
				[11]

3.	(a)	<u>4, 4.84, 7.06</u>	B2/1/0	2
	(b)	$I \approx \frac{1}{2} \times 0.25\,[6.06 + 7.06 + 2(4.32 + 4 + 4.84)]$	B1 [M1 A1 ft]	
		$= \frac{1}{2} \times 0.25[39.44]$		
		$=$ <u>4.93</u> or <u>4.9</u> (AWRT 4.93 or just 4.9)	A1	4
		B1 $\frac{1}{2} \times 0.25$		
		M1 A1 ft [ ]		
	(c)	$\int_{0.5}^{1.5}\left(\frac{3}{x} + x^4\right)\mathrm{d}x = \left[3\ln x + \frac{1}{5}x^5\right]_{0.5}^{1.5}$	M1 A1	
		$= \left(3\ln 1.5 + \frac{1}{5}1.5^5\right) - \left(3\ln 0.5 + \frac{1}{5}0.5^5\right)$	M1	
		$= \underline{3\ln 3 + 1.5125 \text{ or } 3\ln 3 + \frac{121}{80}}$	A1	4
		M1 Some correct $\int$		
		A1 $3\ln x + \frac{1}{5}x^5$		
		M1 Use of limits		
	(d)	$\frac{[4.93 - (c)]}{(c)} \times 100$, $= 2.53\%$ (i.e. $< 3\%$)		
		AWRT <u>2.5</u>%	M1, A1	2

[12]

1. In part (a), the log working was often unclear and part (b) also gave many difficulty. The differentiation was often incorrect. $\frac{1}{2x}$ was not unexpected but expressions like $x+\frac{1}{x}$ were also seen. Many then failed to substitute $x=\frac{1}{2\text{e}}$ into their $\frac{\text{d}y}{\text{d}x}$ and produced a non-linear tangent. Parts (c) and (d) were well done. A few did, however, give their answers to an inappropriate accuracy. As the table is given to 2 decimal places, the answer should not be given to a greater accuracy.

2. For many candidates this was a good source of marks. Even weaker candidates often scored well in parts (c) and (d). In part (a) there were still some candidates who were confused by the notation, f' often interpreted as f^{-1}, and common wrong answers to the differentiation were $\frac{\text{e}^x}{5}$ and 1 + e^x. The most serious error, which occurred far too frequently, in part (b) was to have a variable gradient, so that equations such as $y-\frac{1}{5}=\left(1+\frac{\text{e}^x}{5}\right)x$ were common. The normal, rather than the tangent, was also a common offering.

3. This question was usually answered well. The calculator work in part (a) caused few problems and using an incorrect number of decimal places was rare. Part (b) was answered well too although there were several instances of h = 0.2 instead of 0.25. A few candidates tested the setter's ingenuity by using ratios 2:1:1:1:2 rather than the correct 1:2:2:2:1 which also yielded an answer of 4.93!

The $\frac{3}{x}$ term in part (c) presented problems for some, and a number ignored the request for an exact answer again. Part (d) was not answered as well as might be expected at this level, far too many candidates had their answer to part (b) on the denominator rather than their answer to part (c).